Interior Angles of Polygons

Where did Poly go? I don't know...

The interior angles of polygons starts with triangles... Oh no, not more triangles... Well, just for an explanation...

Pentagon  Hexagon  Heptagon

How many sides? _______  How many sides? _______  How many sides? _______

How many triangles? _______  How many triangles? _______  How many triangles? _______

Now, if I have a polygon with n (some number of ) sides, how many triangles are in the polygon?

I'll give you this one... it's n-2.

Now look at this one...

Okay, now stay with me here.... The measure of the interior angles of this pentagon is \(\angle EAB + \angle ABC + \angle BCD + \angle CDE + \angle DEA\).

The \(\angle EAB = \angle 1 + \angle 2 + \angle 3\) and \(\angle ABC = \angle 9\),

\(\angle BCD = \angle 7 + \angle 8\), \(\angle CDE = \angle 1 + \angle 2\), and \(\angle DEA = \angle 4\).

So, that means that the sum of the measure of the interior angles of the pentagon is equal to the sum of the measures of the interior angles of all of the triangles! How many triangles are there? n-2 of course! n=5 because that's how many sides there are so there are 5-2 or 3 triangles (as you can see). That means the sum of the interior angles of the pentagon is 3*180°, which is 540°. That gives us this formula...

\[
\text{Sum of the interior angles of any polygon} = (n-2)180°.
\]

Regular polygons.... A regular polygon is actually quite a special thing. The word regular means equiangular and equilateral. Huh? In plain ole' English, that means a regular polygon's angles are all the same measurement, and its sides are all the same length. Like this...

If this polygon's interior angles all add to 540°, how could we find the measure of one angle, say angle A? Divide it by 5! (since there are 5 angles).

\[\angle A = 540° / 5 = 108°\]. So, that leads to a new formula...

One interior angle of a REGULAR polygon with n sides = \(\frac{(n-2)180°}{n}\)

Let's check out some examples...

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Ex. 1. What is the sum of the interior angles of a hexagon?

\[ n=6 \text{ and } (n-2)180° \text{ so } (6-2)180°=4*180°=720° \]

Ex. 2. What is measure of one interior angle of a regular hexagon?

\[ n=6 \text{ and } (n-2)180° \text{ so } (6-2)180°=4*180°=720°=120° \]

Ex. 3. How many sides does a polygon have if the sum of its interior angles = 2,340°?

\[
\begin{align*}
\frac{(n-2)180°}{180} &= 2,340° \\
\frac{(n-2)180°}{180} &= 2,340° \\
\frac{n-2}{180} &= 13 \\
180 &+ 2 + 2 \\
n &= 15 \text{ sides}
\end{align*}
\]

Ex. 4. Find the missing angle...

Step 1. Count the sides...

\[ n=5 \]

Step 2. Find the sum of the interior angles...

\[ (n-2)180° \]

\[ (5-2)180° \]

\[ 3*180° \]

\[ 540° \]

Step 3. Set up an equation and solve...

\[ 94° + 78° + 156° + 91° + 91° + x = 540° \]

\[ 419° + x = 540° \]

\[ -419° \]

\[ -419° \]

\[ x = 121° \]

Ex. 5. Solve for x...

Step 1. Count the sides...

\[ n=6 \]

Step 2. Find the sum of the interior angles...

\[ (n-2)180° \]

\[ (6-2)180° \]

\[ 4*180° \]

\[ 720° \]

\[ x + 7 + 2x - 31 + x + 24 + 2x + x - 13 + 2x + 11 = 720° \]

\[ 9x - 2 = 720° \]

\[ +2 \]

\[ +2° \]

\[ \frac{9x}{q} \]

\[ \frac{722°}{q} \]

\[ x = 80, 22 \]

Ex. 6. Solve for x...

Step 1. Count the sides...

\[ n=5 \]

Step 2. Find the angle measure...

\[ \frac{(n-2)180°}{n} \]

\[ \frac{(5-2)180°}{5} \]

\[ 108° \]

\[ \frac{2x + 8}{108} \]

\[ -8 \]

\[ -8 \]

\[ \frac{2x = 100}{2} \]

\[ \frac{2}{2} \]

\[ x = 50 \]

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Answer each question...

1. What is the sum of the interior angles of an Octagon?

2. What is the sum of the interior angles of a Decagon?

3. What is the sum of the interior angles of a Dodecagon? (12 sides)

4. What is the sum of the interior angles of a Nonagon? (9 sides)

5. What is the sum of the interior angles of a 13-gon?

6. What is the sum of the interior angles of a 19-gon?

7. What is the sum of the interior angles of a quadrilateral?

8. What is the sum of the interior angles of a Heptagon? (7 sides)

Bubble all the correct answers from above. Don’t bubble incorrect answers.

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Answer each question...

9. What is the measure of one interior angle of a regular Octagon?

10. What is the measure of one interior angle of a regular Nonagon?

11. What is the measure of one interior angle of a regular quadrilateral? What other name is there for a regular quadrilateral?

12. What is the measure of one interior angle of a regular 19-gon?

13. What is the measure of one interior angle of a regular Decagon?

14. What is the measure of one interior angle of a regular Heptagon?

15. What is the measure of one interior angle of a regular 11-gon?

16. What is the measure of one interior angle of a regular Dodecagon?

Bubble all the correct answers from above. Don’t bubble incorrect answers.

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17. How many sides does a polygon have if the sum of its interior angles = 900°?

What kind of polygon is this?

19. How many sides does a polygon have if the sum of its interior angles = 3,060°?

What kind of polygon is this?

21. How many sides does a polygon have if the sum of its interior angles = 3,240°?

What kind of polygon is this?

23. How many sides does a polygon have if the sum of its interior angles = 3,780°?

What kind of polygon is this?

18. How many sides does a polygon have if the sum of its interior angles = 1,080°?

What kind of polygon is this?

20. How many sides does a polygon have if the sum of its interior angles = 3,420°?

What kind of polygon is this?

22. How many sides does a polygon have if the sum of its interior angles = 2,160°?

What kind of polygon is this?

24. How many sides does a polygon have if the sum of its interior angles = 2,340°?

What kind of polygon is this?

Bubble all the correct answers from above. Don’t bubble incorrect answers.

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For these... find the missing angle or angles.

25.

26.

27.

28.

29.

30.

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For these... solve for x.

31.

\[ x + 35^\circ = x - 4^\circ \]
\[ x - 18^\circ = 2x + 1^\circ \]

33.

\[ 2x - 17^\circ \]

35.

\[ x + 7^\circ \]
\[ x + 10^\circ \]
\[ x + 18^\circ \]
\[ x - 5^\circ \]
\[ x + 11^\circ \]
\[ x + 29^\circ \]
\[ x + 23^\circ \]

32.

\[ 2x + 38^\circ \]

34.

\[ x + 81^\circ \]
\[ 4x - 10^\circ \]
\[ x + 21^\circ \]
\[ x + 35^\circ \]
\[ 3x + 23^\circ \]
\[ 5x - 60^\circ \]

36.

\[ 5x - 61^\circ \]

Bubble all the correct answers from above. Don’t bubble incorrect answers.

\[ \bigcirc 113 \quad \bigcirc 45.29 \quad \bigcirc 47.32 \quad \bigcirc 41 \quad \bigcirc 135.89 \quad \bigcirc 47 \quad \bigcirc 42 \quad \bigcirc 76 \quad \bigcirc 123.5 \quad \bigcirc 110 \]

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