Remote Exterior Angle Theorem

Aka: Another fancy name for an easy shortcut...

Ex. 1. Find the measure of \( \angle x \).

\[
\begin{array}{c}
87^\circ \\
41^\circ \\
x
\end{array}
\]

What??? How am I supposed to do that? It’s way over there. I don’t know how to do that....
Well, to find things that are hard to find work backwards and then forwards. (More on this later.)
I’m sure you’re saying, “Oh yeah! What if I found what \( \angle y \) is first then I could use the linear pair postulate to find \( m\angle x \)?” You’re so smart! So, working forward, find \( m\angle y \) by using the Triangle Sum Theorem.

\[
41^\circ + 87^\circ + y = 180^\circ
\]

Great! Now use the linear pair postulate to find \( x \).

\[
y = \quad \quad \quad \quad x = 128^\circ
\]

Whoa! Wait a minute there! I already know what you’re thinking, “Why didn’t you just tell me that when I was finding angle \( y \)?” If 128° is the answer, “Why can’t I just add 41° + 87° and get the answer??!!” Of course you can! But, would you really have appreciated the Remote Exterior Angle Theorem in all its easiness if I had just told you? Oh, and you really shouldn’t get mad. It’s just Geometry you know...

All right, so it worked once, big deal. How do we know if that will always work? No...don’t wait until the quiz to find out... Let’s prove it now.

Start with this triangle.

\[
\begin{array}{c}
1 \\
2 \\
4 \\
3
\end{array}
\]

1. \( m\angle 1 + m\angle 2 + m\angle 4 = 180^\circ \)  
2. \( m\angle 3 + m\angle 4 = 180^\circ \)  
3. \( m\angle 1 + m\angle 2 + m\angle 4 = m\angle 3 + m\angle 4 \)  
4. \( m\angle 1 + m\angle 2 = m\angle 3 \)

Remote Exterior Angle Theorem: \( m\angle 1 + m\angle 2 = m\angle 3 \)

You might not have to know this to pass a test, but you should read it over and over until it makes sense. That way the Remote Exterior Angle Theorem will make much more sense, and it will really impress that person who you’ve been staring at across the classroom... Maybe not, who knows?

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Okay, so here it is...

**Remote Exterior Angle Theorem:** The sum of the measures of two angles of a triangle is equal to the measure of the Remote Exterior Angle. \( m\angle 1 + m\angle 2 = m\angle 3 \) (memorize me)

![Diagram of a triangle with angles labeled 1, 2, and 3.]

Pay attention so you don't mess up!!!!!

Angle 3 is called "remote" because it’s far away like the remote continent of Antarctica is far away. (That's the one on the bottom of the globe... remember?) IT ONLY WORKS IF THE EXTERIOR (OUTSIDE) ANGLE IS FAR AWAY FROM THE TWO INSIDE ONES.

Got it? Okay, let's practice some. Remember the more you practice the easier the test will be, so no whining! J/K :-) Let's go!

For each find the measure of angle \( x \).

1. [Diagram with angles 114° and 38° and \( x \).]

   **Step 1.** Write out the theorem.
   
   \( m\angle 1 + m\angle 2 = m\angle 3 \)

   **Step 2.** Plug in the given values.
   
   \( 114^\circ + 38^\circ = x^\circ \)

   **Step 3.** Solve.
   
   \( 152^\circ = x \)

2. [Diagram with angles 54° and 63° and \( x \).]

   **Step 1.** Write out the theorem.
   
   **Step 2.** Plug in the given values.
   
   **Step 3.** Solve.

On that last one, don't worry if it's rotated or flipped. The theorem doesn't change. If it helps, you can turn your paper so the figure is "right-side-up." Let's do some more!
3. Step 1. Write out the theorem.
Step 2. Plug in the given values.
Step 3. Solve.

5. Don’t forget to keep showing all the steps!

7.

4. Step 1. Write out the theorem.
Step 2. Plug in the given values.
Step 3. Solve.

6.

8.

Bubble all the correct answers from above. Don’t bubble incorrect answers.

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9. This isn't as difficult as it looks...

\[ m\angle 1 + m\angle 2 = m\angle 3 \]
\[ 29^\circ + x = 87^\circ \]
\[ -29^\circ -29^\circ \]
\[ x = ____ \]

See? Next, you try one...

11.

13.

10. Don't forget to show all the steps, like #9.

12.

14. Don't over-think this one. Just plug those x's in!

Bubble all the correct answers from above. Don't bubble incorrect answers.

\[ \bigcirc 35^\circ \quad \bigcirc 67^\circ \quad \bigcirc 13^\circ \quad \bigcirc 93^\circ \quad \bigcirc 74^\circ \quad \bigcirc 32^\circ \quad \bigcirc 58^\circ \quad \bigcirc 53^\circ \quad \bigcirc 25^\circ \quad \bigcirc 49^\circ \]

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15. Try this on for size!

\[ m\angle 1 + m\angle 2 = m\angle 3 \]

\[ (x+3^\circ) + (x+4^\circ) = 125^\circ \]
\[ x+3^\circ + x+4^\circ = 125^\circ \]
\[ 2x+7^\circ = 127^\circ \]
\[ -7^\circ = -7^\circ \]
\[ 2x = 120^\circ \]
\[ \frac{2}{2} \]
\[ x = 60^\circ \]

16. Now you do one.

17.

\[ 2x + 37^\circ \]
\[ 3x^\circ \]

18.

19. See #14 for help.

\[ 135^\circ \]
\[ x+13^\circ \]

20.

x+34°
3x
21.

22.

23. Try this one. Write the angle measures in the diagram first!

\[ m \angle A = 48^\circ, m \angle B = 93^\circ, \text{ find } m \angle BCD \]

24. \( m \angle T = 35^\circ, m \angle H = 61^\circ. \) Find \( m \angle MAT \)

25. \( m \angle XYW \) is 6 times \( m \angle W, mz = m \angle W + 71^\circ. \) Find \( m \angle Z. \) (Write everything in the diagram first. Let's use \( x \) to represent \( m \angle W? \))

\[ m \angle 1 + m \angle 2 = m \angle 3 \]
\[ (x + 71^\circ) + x = 6x \]

\[ m \angle Z = x + 71^\circ \]
\[ m \angle Z = (\_\_\_) + 71^\circ \]
\[ m \angle Z = \_\_\_ + 71^\circ \]
\[ m \angle Z = \_\_\_ \]

\[ \text{This is your final answer, not what } x \text{ equals.} \]

Bubble all the correct answers from above. Don't bubble incorrect answers.

\[ \_\_\_ \_\_ 45^\circ \_\_\_ 45^\circ \_\_\_ 55^\circ \_\_\_ 78.75^\circ \_\_\_ 29^\circ \_\_\_ 17.75^\circ \_\_\_ 96^\circ \_\_\_ 141^\circ \_\_\_ 39^\circ \_\_\_ 162^\circ \]

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26. $\angle R = 42^\circ$, $\angle CAS$ is $19^\circ$ more than twice $\angle S$. Find $\angle CAS$ (see #25).

27. $\triangle CAT$ is equilateral. Find $\angle ATS$.

28. $\triangle DOG$ is isosceles, $\angle OGS = 148^\circ$. Find $\angle D$.

29. $\angle L$ is $51^\circ$ more than two times $\angle P$, $\angle HEP$ is $9^\circ$ more than four times $\angle P$. Find $\angle HEP$.

30. $\angle GRE$ is $75^\circ$, $\angle EAT$ is $9^\circ$ more than $3$ times $\angle E$. Find $\angle EAT$.

Bubble all the correct answers from above. Don't bubble incorrect answers.

○ 46° ○ 48° ○ 153° ○ 42° ○ 177° ○ 65° ○ 120° ○ 23° ○ 74° ○ 105°

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