Solving Systems of Linear Equations and Circles

Adapted from Walch Education
Important-

- A system of equations is a set of equations with the same unknowns.

- If a two-equation system has a linear equation and a circle equation, then the system can have no real solutions, one real solution, or two real solutions.
Also,

- A solution is an ordered pair; its graphical representation is a point at which the line and circle intersect.

No real solution

One real solution

Two real solutions

6.3.1: Solving Systems of Linear Equations and Circles
Key Concepts

• Using substitution, a system of a linear equation and a circle equation can be reduced to a single quadratic equation whose solutions lead to the solutions of the system.

• The quadratic formula $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$.

6.3.1: Solving Systems of Linear Equations and Circles
Key Concepts, Continued

Every **quadratic equation** of the form $ax^2 + bx + c = 0$ that has one or two real solutions can be solved by the quadratic formula; some can be solved more easily by factoring and using the Zero Product Property.

- To **factor a polynomial** means to write it as a product of two or more polynomials.
- The **Zero Product Property** states that if a product equals 0, then at least one of its factors is 0.

6.3.1: Solving Systems of Linear Equations and Circles
Reminder…

• The distance formula states that the distance between points \((x_1, y_1)\) and \((x_2, y_2)\) is equal to

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]

• The midpoint formula states that the midpoint of the line segment connecting points \((x_1, y_1)\) and \((x_2, y_2)\) is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

6.3.1: Solving Systems of Linear Equations and Circles
Practice

Solve the system below. Check the solution(s), then graph the system on a graphing calculator.

\[
\begin{align*}
4x + 3y &= 25 \\
 x^2 + y^2 &= 25
\end{align*}
\]
Step 1

Solve the linear equation for one of its variables in terms of the other.

\[-4x + 3y = 25\]  
Equation of the line

\[3y = 4x + 25\]  
Add 4x to both sides to isolate the y-term on the left side.

\[y = \frac{4}{3}x + \frac{25}{3}\]  
Divide both sides by 3 to solve for y in terms of x.
Step 2

Use the result from step 1 to substitute for $y$ in the circle equation.

\[
x^2 + y^2 = 25
\]

Equation of the circle

\[
x^2 + \left(\frac{4}{3} x + \frac{25}{3}\right)^2 = 25
\]

Substitute $\frac{4}{3} x + \frac{25}{3}$ for $y$.

\[
x^2 + \frac{16}{9} x^2 + \frac{200}{9} x + \frac{625}{9} = 25
\]

Square the binomial.

\[
9 x^2 + 16 x^2 + 200 x + 625 = 225
\]

Multiply both sides by 9 to eliminate fractions.

6.3.1: Solving Systems of Linear Equations and Circles
Step 2, continued

25x^2 + 200x + 625 = 225

Combine like terms.

25x^2 + 200x + 400 = 0

Subtract 225 from both sides to get 0 on one side.

x^2 + 8x + 16 = 0

Divide both sides by 25 to simplify.

(x + 4)(x + 4) = 0

Factor.

x + 4 = 0 and x + 4 = 0

Set the factors equal to 0.

x = -4 and x = -4

Solve.

6.3.1: Solving Systems of Linear Equations and Circles
Step 3

Substitute the x-value from step 2 into the linear equation and find the corresponding y-value.

\[ y = \frac{4}{3} x + \frac{25}{3} \]

Linear equation solved for y

Substitute \(-4\) for x.

\[ y = \frac{4}{3} ( -4 ) + \frac{25}{3} \]

Simplify, then solve.

\[ y = \frac{16}{3} + \frac{25}{3} \]
Step 3, continued

\[ y = \frac{9}{3} \]
\[ y = 3 \]

- The solution to the system of equations is \((-4, 3)\).
Step 4

Check the solution by substituting it into both original equations.

Equation of the line: 
\[-4x + 3y = 25\]
\[-4(-4) + 3(3) = 25\]
\[16 + 9 = 25\]
\[25 = 25\]

Equation of the circle: 
\[x^2 + y^2 = 25\]
\[(-4)^2 + 3^2 = 25\]
\[16 + 9 = 25\]
\[25 = 25\]

The solution checks. The solution of the system is \((-4, 3)\).
Step 5

Graph the system on a graphing calculator.

- First, solve the circle equation for $y$ to obtain functions that can be graphed.
  \[ y = \pm \sqrt{25 - x^2} \]

- The linear equation solved for $y$ is
  \[ y = \frac{4}{3} x + \frac{25}{3} \]
The Graph
See if you can...

Determine whether the line with equation $x = -5$ intersects the circle centered at $(-3, 1)$ with radius 4. If it does, then find the coordinates of the point(s) of intersection.
Thanks for watching !!!
~dr. dambreville