Building Functions from Context

Adapted from Walch Education
Key Concepts

- The vertex of a parabola represents either the maximum or minimum $y$-value for the equation.
- The vertex of a parabola can be found using $\left( \frac{-b}{2a}, f \left( \frac{-b}{2a} \right) \right)$ from the general form of the quadratic equation, where $f(x) = ax^2 + bx + c$.
- The vertex of a parabola can be found using $(h, k)$ in vertex form, where $f(x) = a(x - h)^2 + k$. 
Key Concepts

• The **curve** of a parabola is the graphical representation of the solution set for the equation of the parabola.

• A quadratic equation can be built using the vertex and any other point on the parabola.

• Concavity refers to the direction the parabola faces.

• The vertex of an upward-facing (or concave up) parabola will be at the bottom or minimum of the curve.

• Conversely, the vertex of a downward-facing (or concave down) parabola will be at the top or maximum of the parabola.
Concavity

Concave Up

The vertex is a minimum.

Concave Down

The vertex is a maximum.
Key concepts, continued

- The leading coefficient is the coefficient of the term with the highest power.
  - The leading coefficient of the equation of a parabola determines its concavity.
    - If the leading coefficient is positive, the parabola is concave up and the graph has a minimum.
    - If the leading coefficient is negative, the parabola is concave down and the graph has a maximum.
A farmer is building a rectangular pen using 100 feet of electric fencing and the side of a barn. In addition to fencing, there will be a 4-foot gate also requiring the electric fencing on either side of the pen. The farmer wants to maximize the area of the pen. How long should he make each side of the fence in order to create the maximum area?
Write the expressions that describes the length of each side of the pen

- Starting with one of the sides that is perpendicular to the barn, we can say the length is an unknown amount, \( x \), plus the 4-foot gate, or \( x + 4 \).

- The length of the second side of the fence that is perpendicular to the barn will be the same length as the first.
Continued,

- The side that is parallel to the barn is whatever amount of fence is left over after creating the two perpendicular sides.

- The total amount of fencing is 100 feet, and there are two sides of length \((x + 4)\), so the length of the side that is parallel to the barn is \(100 - 2(x + 4)\).

- Simplifying the expression, we get \(100 - 2x - 8\), or \(92 - 2x\).
Build the equation that described the area of the pen

- Remember that area equals length times width, or \( A = l \cdot w \).
- Let \( l = x + 4 \) and \( w = 92 - 2x \).

\[
A(x) = (x + 4)(92 - 2x)
\]

Substitute values for length and width.

\[
A(x) = 92x - 2x^2 + 368 - 8x
\]

Multiply.

\[
A(x) = -2x^2 + 84x + 368
\]

Reorder and simplify.
To find the maximum area, use the vertex

- Note that the leading coefficient of the equation is negative. This means the graph of the equation will be a downward-facing parabola. Therefore, the vertex of the parabola will describe the greatest amount of area.

\[ A(x) = -2x^2 + 84x + 368 \]

- Given that the general form of a quadratic function is \( y = ax^2 + bx + c \), we can determine that \( a = -2 \), \( b = 84 \), and \( c = 368 \).
Continued,

- Find the $x$-coordinate of the vertex by substituting in $a$ and $b$ values from the quadratic function into the expression $\frac{b}{2a}$:

\[
\frac{b}{2a} = \frac{(84)}{2(2)} = \frac{84}{4} = 21
\]

The $x$-coordinate of the vertex is 21.
Continued,

- Find the $y$-coordinate of the vertex by substituting the $x$-coordinate from the vertex into the quadratic function.

\[
A(x) = -2x^2 + 84x + 368 \quad \text{Quadratic function}
\]

\[
A(21) = -2(21)^2 + 84(21) + 368 \quad \text{Substitute 21 for } x.
\]

\[
A(21) = -882 + 1764 + 368 \quad \text{Simplify, then solve.}
\]

\[
A(21) = 1250
\]

- The maximum area of the pen is 1,250 ft$^2$. 

5.7.1: Building Functions from Context
Finally, use the x-value from the vertex to find the lengths of each side of the pen

\[ x = 21 \]

- Each side that is perpendicular to the barn is equal to \( x + 4 \).
  
  \[(x + 4) = (21 + 4) = 25 \text{ feet}\]

- The side that is parallel to the barn is equal to \( 92 - 2x \).
  
  \[(92 - 2x) = [92 - 2(21)] = 50 \text{ feet}\]
Thanks for Watching!

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