Georgia Standards of Excellence Framework

Accelerated GSE Algebra I/Geometry A • Unit 2

Mathematics

Accelerated GSE Algebra I/Geometry A
Unit 2: Reasoning with Linear Equations and Inequalities
OVERVIEW

In this unit students will:

- Solve linear equations in one variable.
- Justify the process of solving an equation.
- Rearrange formulas to highlight a quantity of interest.
- Solve linear inequalities in one variable.
- Solve a system of two equations in two variables by using multiplication and addition.
- Justify the process of solving a system of equations.
- Solve a system of two equations in two variables graphically.
- Graph a linear inequality in two variables.
- Analyze linear functions using different representations.
- Interpret linear functions in context.
- Investigate key features of linear graphs.
- Recognize arithmetic sequences as linear functions.

By the end of eighth grade students have had a variety of experiences creating equations. In this unit, students continue this work by creating equations to describe situations. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to rearrange formulas to highlight a quantity of interest, analyze and explain the process of solving an equation, and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. Students explore systems of equations, find, and interpret their solutions. Students create and interpret systems of inequalities where applicable. For example, students create a system to define the domain of a particular situation, such as a situation limited to the first quadrant. The focus is not on solving systems of inequalities. Solving systems of inequalities can be addressed in extension tasks.

All this work is grounded on understanding quantities and on relationships between them. In earlier grades, students define, evaluate, and compare functions and use them to model relationships between quantities. In this unit, students expand their prior knowledge of functions, learn...
function notation, develop the concepts of domain and range, analyze linear functions using different representations, and understand the limitations of various representations. Students investigate key features of linear graphs and recognize arithmetic sequences as linear functions. Some standards are repeated in units 3, 4, and 5 as they apply to quadratics and exponentials.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the “Strategies for Teaching and Learning” in the Comprehensive Course Overview and the tasks listed under “Evidence of Learning” be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Create equations that describe numbers or relationships

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which A = P(1 + r/n)^nt has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.
MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm’s law \( V = IR \) to highlight resistance \( R \); Rearrange area of a circle formula \( A = \pi r^2 \) to highlight the radius \( r \).

Understand solving equations as a process of reasoning and explain the reasoning
MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

Solve equations and inequalities in one variable
MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given \( ax + 3 = 7 \), solve for \( x \).

Solve systems of equations
MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically
MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation \( f(x) = g(x) \) is the \( x \)-value where the \( y \)-values of \( f(x) \) and \( g(x) \) are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Build a function that models a relationship between two quantities
MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. For example, if Jimmy starts out with \$15 \) and earns \$2 \) a day, the explicit expression “\( 2x + 15 \)” can be described recursively (either in writing or verbally) as “to find out how much money Jimmy will have tomorrow, you add \$2 \) to his total today.” \( J_n = J_{n-1} + 2, J_0 = 15 \)
MGSE9-12.F.BF.2 Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Understand the concept of a function and use function notation
MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1=7$, $a_n=a_{n-1}+2$; the sequence $s_n = 2(n-1) + 7$; and the function $f(x) = 2x + 5$ (when x is a natural number) all define the same sequence.

Interpret functions that arise in applications in terms of the context
MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.
MGSE9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
ENDURING UNDERSTANDINGS

- Create linear equations and inequalities in one variable and use them in a contextual situation to solve problems.
- Create equations in two or more variables to represent relationships between quantities.
- Rearrange formulas to highlight a quantity of interest.
- Solve linear equations and inequalities in one variable.
- Graph linear equations and inequalities in two variables.
- Linear equations and inequalities can be represented graphically and solved using real-world context.
- Solve systems of linear equations in two variables exactly and approximately and explain why the elimination method works to solve a system of two-variable equations.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Understand the concept of a function and be able to use function notation.
- Understand how to interpret linear functions that arise in applications in terms of the context.
- When analyzing linear functions, different representations may be used based on the situation presented.
- A function may be built to model a relationship between two quantities.
- Understand and interpret key features of functions.
- Understand how to interpret expressions for functions in terms of the situation they model.
- Understand that sequences are functions.
Write recursive and explicit formulas for arithmetic sequences and understand the appropriateness of the use of each.
SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Algebra**: The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.

- **Arithmetic Sequence**: A sequence of numbers in which the difference between any two consecutive terms is the same.

- **Average Rate of Change**: The change in the value of a quantity by the elapsed time. For a function, this is the change in the y-value divided by the change in the x-value for two distinct points on the graph.

- **Coefficient**: A number multiplied by a variable in an algebraic expression.

- **Constant Rate of Change**: With respect to the variable x of a linear function y = f(x), the constant rate of change is the slope of its graph.
• **Continuous.** Describes a connected set of numbers, such as an interval.

• **Discrete.** A set with elements that are disconnected.

• **Domain.** The set of x-coordinates of the set of points on a graph; the set of x-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

• **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.

• **Equation:** A number sentence that contains an equals symbol.

• **Explicit Formula.** A formula that allows direct computation of any term for a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$.

• **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

• **Expression.** Any mathematical calculation or formula combining numbers and/or variables using sums, differences, products, quotients including fractions, exponents, roots, logarithms, functions, or other mathematical operations.

• **Factor.** For any number $x$, the numbers that can be evenly divided into $x$ are called factors of $x$. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.

• **Inequality:** Any mathematical sentence that contains the symbols $>$ (greater than), $<$ (less than), $\leq$ (less than or equal to), or $\geq$ (greater than or equal to).

• **Interval Notation.** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.

• **Linear Function.** A function with a constant rate of change and a straight line graph.
• **Linear Model.** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

• **Ordered Pair:** A pair of numbers, (x, y), that indicate the position of a point on a Cartesian plane.

• **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.

• **Range.** The set of all possible outputs of a function.

• **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of \( a_n \).

• **Slope.** The ratio of the vertical and horizontal changes between two points on a surface or a line.

• **Substitution:** To replace one element of a mathematical equation or expression with another.

• **Term.** A value in a sequence—the first value in a sequence is the 1\(^{\text{st}}\) term, the second value is the 2\(^{\text{nd}}\) term, and so on; a term is also any of the monomials that make up a polynomial.

• **Variable:** A letter or symbol used to represent a number.

• **X-intercept.** The point where a line meets or crosses the x-axis

• **Y-intercept.** The point where a line meets or crosses the y-axis

**The Properties of Operations**

Here \( a, b \) and \( c \) stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

- **Associative property of addition** \( (a + b) + c = a + (b + c) \)
- **Commutative property of addition** \( a + b = b + a \)
- **Additive identity property of 0** \( a + 0 = 0 + a = a \)
Existence of additive inverses
For every $a$ there exists $-a$ so that $a + (-a) = (-a) + a = 0$.

Associative property of multiplication
$(a \times b) \times c = a \times (b \times c)$

Commutative property of multiplication
$a \times b = b \times a$

Multiplicative identity property of 1
$a \times 1 = 1 \times a = a$

Existence of multiplicative inverses
For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.

Distributive property of multiplication over addition
$a \times (b + c) = a \times b + a \times c$

The Properties of Equality
Here $a$, $b$ and $c$ stand for arbitrary numbers in the rational, real, or complex number systems.

- Reflexive property of equality
  $a = a$

- Symmetric property of equality
  If $a = b$, then $b = a$.

- Transitive property of equality
  If $a = b$ and $b = c$, then $a = c$.

- Addition property of equality
  If $a = b$, then $a + c = b + c$.

- Subtraction property of equality
  If $a = b$, then $a - c = b - c$.

- Multiplication property of equality
  If $a = b$, then $a \times c = b \times c$.

- Division property of equality
  If $a = b$ and $c \neq 0$, then $a ÷ c = b ÷ c$.

- Substitution property of equality
  If $a = b$, then $b$ may be substituted for $a$ in any expression containing $a$. 
Methods for Solving Systems of Equations

Graphing

Substitution

Elimination

Intersecting Lines:
Parallel Lines:
Coinciding Lines:

What if both of the variables cancel out? Look at the resulting arithmetic equation.
* False statement indicates the lines are parallel so _________________.
* True statement indicates the lines coincide so _________________.

Adapted from Graphic Organizer by Dale Graham and Linda Meyer Thomas County Central High School; Thomasville GA

2x + 3y = 5
4x − y = 17
Acting Out (Scaffolding Task)

Name_________________________________ Date__________________

Adapted from Shell Center Leaky Faucet Short Cycle Task

Mathematical Goals
- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions
- How do I choose and interpret units consistently in formulas?
- How can I model constraints using mathematical notation?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

MGSE9–12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

RELATED STANDARDS
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
   a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
   b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
   c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.
MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. For example, money situations are generally reported to the nearest cent (hundredth). Also, an answer’s precision is limited to the precision of the data given.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
Acting Out (Scaffolding Task)

Name_________________________________ Date__________________

Adapted from Shell Center Leaky Faucet Short Cycle Task

Part I:

Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

1. On the given grid:
   a. pick a point to represent the location of the theater.
   b. Illustrate all of the possible places that Erik could live on the grid paper.
   c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.

2. What is the smallest distance, \( d \), that could separate their homes? How did you know?

3. What is the largest distance, \( d \), that could separate their homes? How did you know?

4. Write and graph an inequality in terms of \( d \) to show their boss all of the possible distances that could separate the homes of the 2 actors.
Part II:

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.
Lucy’s Linear Equations and Inequalities (Practice Task)

Name_________________________________ Date________________

Mathematical Goals
• Create one–variable linear equations and inequalities from contextual situations.
• Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions
• How do I interpret parts of an expression in terms of context?
• How do I create equations and inequalities in one variable and use them to solve problems arising from linear functions?
• How can I write, interpret and manipulate algebraic expressions, equations and inequalities?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9–12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

RELATED STANDARDS
MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MGSE9–12.A.SSE.1b Given situation which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

STANDARDS FOR MATHEMATICAL PRACTICE
2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.
Lucy’s Linear Equations and Inequalities (Practice Task)

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch (if necessary)
- Defining a Variable
- Setting up an equation or inequality
- Solve the equation or inequality
- Make sure you answer the question

1. The sum of 38 and twice a number is 124. Find the number.

2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.

4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?
5. Find three consecutive integers whose sum is 171.

6. Find four consecutive even integers whose sum is 244.

7. Alex has twice as much money as Jennifer. Jennifer has $6 less than Shannon. Together they have $54. How much money does each have?

8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?
Jaden’s Phone Plan (Scaffolding Task)

Name_________________________________ Date________________

Mathematical Goals
• Create one–variable linear equations and inequalities from contextual situations.
• Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions
• How do I solve an equation in one variable?
• How do I justify the solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9–12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. (Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (For example, given ax + 3 = 7, solve for x.)

STANDARDS FOR MATHEMATICAL PRACTICE
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
Scaffolding Task: Jaden’s Phone Plan

Name_________________________________ Date__________________

Jaden has a prepaid phone plan (Plan A) that charges 15 cents for each text sent and 10 cents per minute for calls.

1. If Jaden uses only text, write an equation for the cost $C$ of sending $t$ texts.

   a. How much will it cost Jaden to send 15 texts? Justify your answer.
   
   b. If Jaden has $6, how many texts can he send? Justify your answer.

2. If Jaden only uses the talking features of his plan, write an equation for the cost $C$ of talking $m$ minutes.

   a. How much will it cost Jaden to talk for 15 minutes? Justify your answer.
   
   b. If Jaden has $6, how many minutes can he talk? Justify your answer.

3. If Jaden uses both talk and text, write an equation for the cost $C$ of sending $t$ texts and talking $m$ minutes.

   a. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.
   
   b. If Jaden wants to send 21 texts and only has $6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.
Jaden discovers another prepaid phone plan (Plan \( B \)) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

4. Write an equation for the cost of Plan \( B \).

In an average month, Jaden sends 200 texts and talks for 100 minutes.

5. Which plan will cost Jaden the least amount of money? Justify your answer.
Scaffolding Task: Forget the Formula

Name_____________________________ Date________________

Mathematical Goals
- Rearrange formulas to highlight a quantity of interest.
- Create equations in two variables to represent relationships.
- Write and graph an equation to represent a linear relationship.
- Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

Essential Questions
- How do I interpret parts of an expression in terms of context?
- How do I create equations in two variables to represent relationships between quantities?
- How can I rearrange formulas to highlight a quantity of interest?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which \( A = P(1 + r/n)^nt \) has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm’s law \( V = IR \) to highlight resistance \( R \); Rearrange area of a circle formula \( A = \pi r^2 \) to highlight the radius \( r \).

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
Forget the Formula (Scaffolding Task)

Temperature can be measured with many different systems, the most commonly used are Fahrenheit and Celsius. The relationship between the two systems is linear and therefore can be determined using any two equivalent measurements.

1. What is the boiling point of water in Fahrenheit and in Celsius?

2. What is the freezing point of water in Fahrenheit and in Celsius?

3. Using these two points, create an equation that convert Celsius to Fahrenheit.
4. Rearrange the equation found in the previous problem to solve for Celsius.

5. What does the constant represent in each equation? What does the slope represent in each equation?

6. At what temperature is the degrees Celsius equal to the degrees Fahrenheit?
Cara’s Candles Revisited (Scaffolding Task)

Name_________________________ Date__________________

Mathematical Goals

- Determine whether a point is a solution to an equation.
- Determine whether a solution has meaning in a real-world context.
- Interpret whether the solution is viable from a given model.
- Write and graph equations and inequalities representing constraints in contextual situations.

Essential Questions

- How do I graph equations on coordinate axes with the correct labels and scales?
- How do I create equations in two or more variables to represent relationships between two quantities?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which \( A = P(1 + r/n)^{nt} \) has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation \( f(x) = g(x) \) is the x-value where the y-values of \( f(x) \) and \( g(x) \) are the same.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.
Cara’s Candles Revisited (Scaffolding Task)

Name_________________________________ Date__________________

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. You will explain your thinking using the table and create a graphical representation of the situation.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>16 cm candle height (cm)</th>
<th>12 cm candle height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>13.5</td>
<td>10.5</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Complete the table, and use it to write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.
2. Create a graphical representation of your data, taking into account natural restrictions on domain, range, etc.

3. Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

4. If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.
Solving Systems of Equations Algebraically (Scaffolding Task)

Name_________________________________ Date__________________

Introduction
In this task, students justify the solution to a system of equations by both graphing and substituting values into the system. Students will then show that multiplying one or both equations in a system of equations by a constant creates a new system with the same solutions as the original. This task will lead into using the elimination method for solving a system of equations algebraically.

Mathematical Goals
- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions
- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
Solving Systems of Equations Algebraically (Scaffolding Task)

Name_________________________________ Date________________

Part 1:

You are given the following system of two equations:

\[ x + 2y = 16 \]
\[ 3x - 4y = -2 \]

1. What are some ways to prove that the ordered pair \((6, 5)\) is a solution?

   a. Prove that \((6, 5)\) is a solution to the system by graphing the system.

   b. Prove that \((6, 5)\) is a solution to the system by substituting in for both equations.

2. Multiply both sides of the equation \(x + 2y = 16\) by the constant ‘7’. Show your work.

\[ 7(x + 2y) = 7 \times 16 \]
a. Does the new equation still have a solution of (6, 5)? Justify your answer.

b. Why do you think the solution to the equation never changed when you multiplied by the ‘7’?

3. Did it have to be a ‘7’ that we multiplied by in order for (6, 5) to be a solution?

   a. Multiply \( x + 2y = 16 \) by three other numbers and see if (6, 5) is still a solution.

      i. ______________________

      ii. ______________________

      iii. ______________________

   b. Did it have to be the first equation \( x + 2y = 16 \) that we multiplied by the constant for (6, 5) to be a solution? Multiply \( 3x - 4y = -2 \) by ‘7’? Is (6, 5) still a solution?

   c. Multiply \( 3x - 4y = -2 \) by three other numbers and see if (6, 5) is still a solution.

      i. ______________________

      ii. ______________________

      iii. ______________________
4. Summarize your findings from this activity so far. Consider the following questions:
   What is the solution to a system of equations and how can you prove it is the solution?
   Does the solution change when you multiply one of the equations by a constant?
   Does the value of the constant you multiply by matter?
   Does it matter which equation you multiply by the constant?

Let’s explore further with a new system.  
\[5x + 6y = 9\]
\[4x + 3y = 0\]

5. Show by substituting in the values that \((-3, 4)\) is the solution to the system.

6. Multiply \(4x + 3y = 0\) by ‘\(-5\)’. Then add your answer to \(5x + 6y = 9\). Show your work below.

\[(-5)*(4x + 3y) = (-5)*0 \quad \Rightarrow \quad \text{________________________ Answer} \]
\[+ \quad 5x + 6y = 9 \quad \text{________________________ New Equation} \]

7. Is \((-3, 4)\) still a solution to the new equation? Justify your answer.

8. Now multiply \(4x + 3y = 0\) by ‘\(-2\)’. Then add your answer to \(5x + 6y = 9\). Show your work below.
a. What happened to the y variable in the new equation?

b. Can you solve the new equation for x? What is the value of x? Does this answer agree with the original solution?

c. How could you use the value of x to find the value of y from one of the original equations? Show your work below.

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

9. \(-3x + 2y = -6\) \hspace{1cm} 10. \(-5x + 7y = 11\)
   \hspace{1cm} 5x - 2y = 18 \hspace{1cm} 5x + 3y = 19
Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. \(4x + 3y = 14\) (Equation 1)
2. \(-2x + y = 8\) (Equation 2)

Choose the variable you want to eliminate.

a. To make the choice, look at the coefficients of the \(x\) terms and the \(y\) terms. The coefficients of \(x\) are ‘4’ and ‘–2’. If you want to eliminate the \(x\) variable, you should multiply Equation 2 by what constant?

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the \(x\) variable?

ii. Solve the equation for \(y\). What value did you get for \(y\)?

iii. Now substitute this value for \(y\) in Equation 1 and solve for \(x\). What is your ordered pair solution for the system?

iv. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.
b. The coefficients of $y$ are ‘3’ and ‘1’. If you want to eliminate the $y$ term, you should multiply Equation 2 by what constant?

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the $y$ variable?

ii. Solve the equation for $x$. What value did you get for $x$?

iii. Now substitute this value for $x$ in Equation 1 and solve for $y$. What is your ordered pair solution for the system?

Use your findings to answer the following in sentence form:

c. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.

d. Would you need to eliminate both variables to solve the problem? Justify your answer.

e. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?
f. How do you decide what constant to multiply by in order to make the chosen variable eliminate?

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

2. 3x + 2y = 6
   \(-6x - 3y = -6\)

3. \(-6x + 5y = 4\)
   7x - 10y = -8

4. 5x + 6y = -16
   2x + 10y = 5
Summer Job (Scaffolding Task)

Mathematical Goals
- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Solve word problems involving inequalities.
- Represent constraints with inequalities.

Essential Questions
- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9–12.A.REI.12  Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE
2. Reason abstractly and quantitatively.
4. Model with mathematics.
Summer Job (Scaffolding Task)

Name_________________________________ Date__________________

In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn $10 per hour while babysitting and $20 per hour while cleaning houses. You need to earn at least $1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

2. Write an expression to represent the amount of money earned while cleaning houses.

3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

4. Graph the mathematical model. Graph the hours babysitting on the x–axis and the hours cleaning houses on the y–axis.
5. Use the graph to answer the following:
   a. Why does the graph only fall in the 1st Quadrant?

   b. Is it acceptable to earn exactly $1000? What are some possible combinations of outcomes that equal exactly $1000? Where do all of the outcomes that total $1000 lie on the graph?

   c. Is it acceptable to earn more than $1000? What are some possible combinations of outcomes that total more than $1000? Where do all of these outcomes fall on the graph?

   d. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

6. How would the model change if you could only earn more than $1000? Write a new model to represent needing to earn more than $1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.
You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

7. Write a mathematical model representing the amount of money spent on jeans and shirts.

8. Graph the mathematical model.
   Graph the number of jeans on the x–axis and shirts on the y–axis.
   a. Why does the graph only fall in the 1st Quadrant?
   b. Is it acceptable to spend less than $400? What are some possible combinations of outcomes that total less than $400? Where do all of these outcomes fall on the graph?
   c. Is it acceptable to spend exactly $400? How does the graph show this?
   d. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?
Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

9. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?

10. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?
Extension Task: Graphing Inequalities

Name_________________________________ Date________________

Mathematical Goals
• Solve word problems involving inequalities.
• Represent constraints with inequalities.
• Rearrange and graph inequalities.

Essential Questions
• How do I graph a linear inequality in two variables?
• How do I justify a solution to an equation?
• How do I graph a system of linear inequalities in two variables.

GEORGIA STANDARDS OF EXCELLENCE
MGSE9–12.A.REI.12  Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.
Graphing Inequalities (Extension Task)

Name_________________________________ Date__________________

1. Graph the inequality $y > -\frac{1}{2}x + 5$. What are some solutions to the inequality?

2. Graph the inequality $y < x + 2$. What are some solutions to the inequality?
3. Look at both graphs.
   
   a. Are there any solutions that work for both inequalities? Give 3 examples.
   
   b. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.

4. Graph both inequalities on the same coordinate system, using a different color to shade each.

   a. Look at the region that is shaded in both colors. What does this region represent?

   b. Look at the regions that are shaded in only 1 color. What do these regions represent?
c. Look at the region that is not shaded. What does this region represent?

5. Graph the following system on the same coordinate grid. Use different colors for each.

\[ \begin{align*}
    x + y &\geq 3 \\
    y &\leq -x + 5
\end{align*} \]

a. Give 3 coordinates that are solutions to the system.

b. Give 3 coordinates that are not solutions to the system.

c. Is a coordinate on either line a solution?

d. How would you change the inequality \( x + y \geq 3 \) so that it would shade below the line?
e. How would you change the inequality $y \leq -x + 5$ so that it would shade above the line?

6. Graph the new equations from ‘d’ and ‘e’ above on the same coordinate grid. Use blue for one graph and red for the other.

a. What do the coordinates in blue represent?

b. What do the coordinates in red represent?

c. Why do the colors not overlap this time?

Graph the following on the same coordinate grid and give 3 solutions for each.

7. $2x + 3y < 6$
   $x + 5y > 5$
8. \[ y \geq \frac{1}{2} x - 1 \]
\[ y \leq -\frac{1}{4} x + 6 \]

9. \[ 3x - 4y > 5 \]
\[ y > \frac{3}{4} x + 1 \]
Family Outing (Performance Task)

Name_________________________________ Date________________

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Represent and solve word problems and constraints using inequalities.

Essential Questions

- How do I graph linear inequalities?
- How do I solve a system of linear equations graphically or algebraically?
- How do I justify the solution to a system of equations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given \( ax + 3 = 7 \), solve for \( x \).

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Family Outing (Performance Task)

Name_________________________________ Date________________

You and your family are planning to rent a van for a 1 day trip to Family Fun Amusement Park in Friendly Town. For the van your family wants, the Wheels and Deals Car Rental Agency charges $25 per day plus 50 cents per mile to rent the van. The Cars R Us Rental Agency charges $40 per day plus 25 cents per mile to rent the same type van.

1. Write a mathematical model to represent the cost of renting a van from the Wheels and Deals Agency for 1 day.

   a. Do the units matter for this equation?

   b. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.

2. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.

   a. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?

   b. Use the equation from ‘2a’ to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.
3. Graph the 2 models on the same coordinate system. Be sure to extend the lines until they intersect.

a. Where do the 2 lines intersect?

b. What does the point of intersection represent?

c. When is it cheaper to rent from Wheels and Deals?

d. When is it cheaper to rent from Cars R Us?

4. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

When you leave the car rental agency, your father goes to the Fill ‘er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

5. If your father spends $78 on gas, approximately how many gallons did he purchase?
While in the store, your father purchases drinks for the six people in your van. Part of your family wants coffee and the rest want a soda.

6. Coffee in the store costs $0.49 per cup and sodas are $1.29 each. The cost of the drinks before tax was $6.14.

   a. Write a mathematical model that represents the total number of cups of coffee and sodas.

   b. Write a mathematical model that represents the cost of the coffee and soda.

   c. Solve the system of equations using the elimination method.

EXTENSION:
When you arrive in Friendly Town at the Family Fun Amusement Park, the 6 people in your family pair up to enter the park. You and your brother decide to enter and ride together. The cost to enter the park is $10, with each ride costing $2.

7. You bring $55 to the park. You must pay to enter the park and you budget an additional $10 for food. Write and solve an inequality to determine the maximum number of rides you can ride. Explain your answer.

8. Your brother brings $70 to the park and budgets $12 for food. How many more rides can he ride than you? Explain your answer.

Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for $2.50 per bag. Quick Eats has cotton candy for $4 per bag and popcorn for $2 per bag.

9. If you use the $10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.

10. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.
11. Graph the system of inequalities. Give two combinations that work for both vendors.

12. Assuming you purchase at least one of each, what is the maximum number of bags of cotton candy and popcorn that work for both equations?

When you leave the park, your father notices that you have used $\frac{3}{4}$ of the tank of gas you purchased before you left.

13. Do you have enough gas to get home? Justify your answer.

14. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.
Functioning Well (Practice Task)

Name_________________________ Date________________

Mathematical Goals
- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Essential Questions
- How do I represent real life situations using function notation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

STANDARDS FOR MATHEMATICAL PRACTICE
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
Functioning Well (Practice Task)

Name_____________________________ Date________________

Consider the definition of a function (A function is a rule that assigns each element of set A to a unique element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

Part I – Function or Not

Determine whether or not each of the following is a function or not. Write “function” or “not a function” and explain why or why not.

<table>
<thead>
<tr>
<th>Relation</th>
<th>Answer and Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
</tr>
</tbody>
</table>
4. \((x, y) = \text{(student’s name, student’s shirt color)}\)

Part II – Function Notation
Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let \(p = f(E)\) where \(p\) is the pounds of fish needed and \(E\) is the expected number of customers.

5. What would the expressions \(f(E + 15)\) and \(f(E) + 15\) mean?

1. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

2. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.

8. The owner of the restaurant planned to host his 2 fish-loving parents in addition to his expected customers for dinner at the restaurant. Illustrate using function notation
Part III – Graphs are Functions
Write each of the points using function notation.

9.

\[ f(n) = 2n \]

\begin{figure}
\begin{center}
\includegraphics[width=0.5\textwidth]{graph.png}
\end{center}
\end{figure}
Putting the “Fun” in Functions (Culminating Task)

Name_________________________________ Date__________________

Mathematical Goals
- Interpret linear models that represent real-life situations
- Understand the concept of a function and use function notation
- Analyze functions using different representations
- Building new functions from existing functions
- Construct and compare linear models and solve problems

Essential Questions
- How can I use and apply what I have learned about linear functions?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^nt$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm’s law $V = IR$ to highlight resistance $R$; Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius $r$.

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given $ax + 3 = 7$, solve for $x$.

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.
MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression “2x+15” can be described recursively (either in writing or verbally) as “to find out how much money Jimmy will have tomorrow, you add $2 to his total today.”

\[ J_n = J_{n-1} + 2, J_0 = 15 \]

MGSE9-12.F.BF.2 Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4,...) By graphing or calculating terms, students should be able to show how the recursive sequence \[ a_1=7, a_n=a_{n-1} + 2; \] the sequence \[ s_n = 2(n-1) + 7; \] and the function \[ f(x) = 2x + 5 \] (when x is a natural number) all define the same sequence.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing,
decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Putting the “Fun” in Functions (Culminating Task)

Name_________________________________ Date__________________

In this unit you have learned the concept of a function and how to use function notation, interpret functions that arise in applications in terms of the context, analyze functions using different representations, building new functions from existing functions, and construct and compare linear models and solve problems.

Using the guide provided, you will construct a function booklet or create a webpage for students who will learn about linear functions next year. Before designing your booklet or webpage, use the guide to plan your pages or links. Make sure you use the graphing calculator to test all of your models prior to adding them to the booklet or webpage. Use the checklist to ensure that all parts of the task have been addressed.

Booklet/ Webpage Planning Guide and Checklist

☐ Booklet Cover/Home link on webpage: (1 point)
  ☐ Give your booklet/page a title
  ☐ Use a mathematical symbol or symbols that are unique to learning about linear functions on your cover or home link
  ☐ Include your name, date, and class period

☐ Table of Contents Page or Link: (1 point)
  ☐ Page number for unit Definitions or link to Definitions
  ☐ Page number or link for Function Notation
  ☐ Page number or link for Interpreting Linear Functions Arising in Applications
  ☐ Page number or link for Analyzing Linear Functions
  ☐ Page number or link for Constructing and Comparing Linear Models
  ☐ Page number or link for Unit Reflection Summary
  ☐ Page number or link for Works Cited

☐ Definitions Page or Link: (8 points)
  ☐ Choose at least 10 important vocabulary words from the unit to define
  ☐ Provide a model or example of each vocabulary word. (You may use symbols, graphs, tables, or pictures.)

☐ Systems of Equations/Inequalities Page or Link: (10 points)
  ☐ Create a contextual situation that would illustrate the application of systems of equations in context. Explain constraints as they apply to the context.
  ☐ Create a contextual situation that would illustrate a linear inequality in two variables. Graph the solution set to the linear inequality.

☐ Function Notation Page or Link: (10 points)
  ☐ Provide at least one example of a domain and range that illustrates a function and explain why it is a function.
  ☐ Provide at least one example of a domain and range that is not a function and explain why.
Create one real world scenario in which function notation may be used to model a linear function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.

Use the scenarios to create a recursive formula

Interpreting Linear Functions Arising in Applications: (20 points)
Create a story that would generate a linear function and describe the meaning of key features (intercepts, intervals where the function is increasing, decreasing, positive, or negative; end behaviors) of the graph as they relate to the story.
Show the graph of your function and relate the domain to the quantitative relationship it describes. Describe the rate of change for a linear function or the rate a change over an interval.

Analyzing Linear Functions: (10 points)
Create one linear function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
Create two different linear functions. Show one algebraically and the other using a verbal description. Compare the two functions.

Building Functions: (10 points)
Explain how to find an explicit expression, a recursive process, or steps for calculation to complete a sequence/pattern. Write the sequence both recursively and with an explicit formula.

Constructing and Comparing Linear Models (20 points)
Design a word problem that involves a linear model. Use a table or sequence to illustrate the relationships described in the models.
Explain the constant rate per unit interval relative to another for the word problem that you designed.
Construct the graphs for each model in the word problem that you designed.
Compare the linear models from your word problem. Interpret the parameters.

Reflection / Summary: (8 points)
Describe your learning journey throughout the unit. Reflect on topics that you found easy to learn and those that were most difficult.
Are there any standards that you need more help grasping? Explain. If not, which standards do you have the best grasp? Explain.
What advice would you give to other students that will learn about linear functions in the future?
Which task(s) did you find the most beneficial to mastering key standards?
Any other insight you would like to share about Unit 2.

Works Cited: (2 points)
Use MLA format to cite any books, websites, and any other references used to create your booklet or webpage.